



## Acceleration Shifts Wavelength

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### Abstract

We show that the acceleration causes redshift, denoted it as “Acceleration-Redshift”. When a wave source is receding with not only velocity but also acceleration, the wavelength is shifted to red. The receding velocity at receiving time is larger than Doppler velocity. The total redshift is a summation of Doppler redshift and Acceleration-Redshift that enlarges continuously Doppler redshift. Moreover we derived General Redshift and General Blueshift formulas that show that not only velocity, but also acceleration, jerk, and higher order derivative shift wavelength. The General Redshift formula is identical with the cosmological redshift, which implies that the cosmological redshift intrinsically includes Acceleration-Redshift. We believe that to describe the accelerating universe, physics laws should include acceleration for accuracy and completeness.

Key words: Doppler effect, redshift, acceleration, cosmological redshift, Blueshift

## 1. Introduction

The classical Doppler effect is a velocity-redshift relation,  $cz = v_{\text{Doppler}}$ , and applied fruitfully in astronomy. In the 1920s, Hubble used Doppler's velocity-redshift relation to measure the speed of stars/galaxies, established the linear distance-redshift and distance-velocity relations, and made a revolutionary discovery that the universe is expanding [1],  $\dot{r} > 0$  and  $\ddot{r} = 0$ . In the 1998, Scientists reported a revolutionary discovery that the expansion of the universe is accelerating [2],  $\ddot{r} > 0$  and  $\dddot{r} = 0$ . In the 2016, Scientists report that even that acceleration is faster than expected [3]. If confirmed, the 2016 observation indicates that the acceleration is accelerating,  $\dddot{r} > 0$ , call it the "jerk".

We believe that to describe the accelerating universe, physics laws, e.g., Hubble law and Doppler effect, should include acceleration for accuracy and completeness.

In the 2017, for explaining the accelerating and jerk universe, Hubble law was extended to include acceleration, jerk, and higher order derivatives [4]. One of the predictions is that acceleration contributes to redshift.

It is nature to expect to extend Doppler's velocity-redshift relation to include acceleration, jerk, and higher order derivatives of distance.

In this article, we derive formulas showing how acceleration, jerk, and higher order derivatives contribute to redshift and blueshift.

## 2. Acceleration Shifts Wavelength

### 2.1. Acceleration-Redshift

Now let's extend Doppler's velocity-redshift relation. Considering a situation that a wave source is receding with not only a velocity  $v_{s1}$  (slower than the speed of wave) but also a constant acceleration  $a_{s1}$  from an observer (Fig. 1).

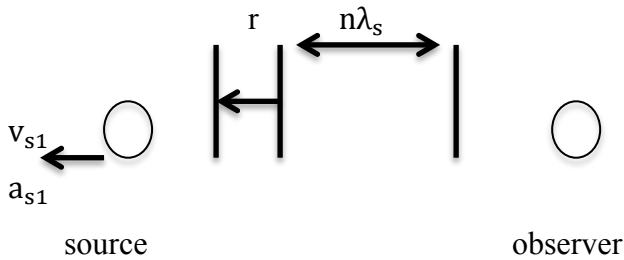


Fig. 1

The mechanism of a constant velocity of source shifts wavelength is that when the wave source is emitting, each successive wave crest is emitted from a position farther

to the observer than the previous wave crest. Therefore, each wave takes longer time to reach the observer than the previous wave, i.e., increasing the wavelength.

For an accelerating source, the same mechanism holds.

The wave velocity is:

$$c = \frac{\lambda_s}{T}. \quad (1)$$

Where  $\lambda_s$  is wavelength and measured in the wave source coordinate system.  $T$  is the time that wave takes to move one wavelength  $\lambda_s$ .

Let's the source emits  $n$  wave crests. Express the emitting time as  $t_1$  and the receiving time as  $t_0$ , then the time interval,

$$nT \equiv t_0 - t_1.$$

During time interval  $nT$ , the source moves a distance  $r$ ,

$$r = v_{s1}(nT) + \frac{1}{2}a_{s1}(nT)^2, \quad (2a)$$

where  $n$  is an integer. Subscript "1" represent the parameter at the emitting time  $t_1$ .

Here we assume a constant acceleration, i.e.,  $a_{s1} = a_{s0}$ . Subscript "0" represent the parameter at the observing time  $t_0$ . Since the source is accelerating, we have:  $v_{s0} > v_{s1}$ . From classical mechanism,  $v_{s1} + a_{s1}(t_0 - t_1) = v_{s0}$ , we re-express equivalently Eq. (2a) as

$$r = v_{s0}(nT) - \frac{1}{2}a_{s0}(nT)^2. \quad (2b)$$

Since the velocity  $v_{s0}$  is faster than the prior velocity  $v_{s1}$ , the distance is equal to velocity  $v_{s0}$  times the time interval, subtracts the distance due to the acceleration.

Substitute Eq. (1) into Eq. (2a) and Eq. (2b) respectively, we obtain,

$$r = v_{s1} \left( n \frac{\lambda_s}{c} \right) + \frac{1}{2} a_{s1} \left( n \frac{\lambda_s}{c} \right)^2. \quad (3a)$$

$$r = v_{s0} \left( n \frac{\lambda_s}{c} \right) - \frac{1}{2} a_{s0} \left( n \frac{\lambda_s}{c} \right)^2. \quad (3b)$$

At time  $t_0$ , the observer receives  $n$  wave crest. The observed wavelength  $\lambda_o$  is longer,

$$n\lambda_o = n\lambda_s + r = n\lambda_s + v_{s1} \left( n \frac{\lambda_s}{c} \right) + \frac{1}{2} a_{s1} \left( n \frac{\lambda_s}{c} \right)^2,$$

or

$$n\lambda_o = n\lambda_s + r = n\lambda_s + v_{s0} \left( n \frac{\lambda_s}{c} \right) - \frac{1}{2} a_{s0} \left( n \frac{\lambda_s}{c} \right)^2.$$

The redshift is

$$z \equiv \frac{\Delta\lambda}{\lambda_s} = \frac{\lambda_o - \lambda_s}{\lambda_s} = \frac{v_{s1}}{c} + \frac{1}{2}a_{s1}n\frac{\lambda_s}{c^2} \quad (4a)$$

or

$$z = \frac{v_{s0}}{c} - \frac{1}{2}a_{s0}n\frac{\lambda_s}{c^2}. \quad (4b)$$

Substituting Eq. (1) into Eq. (4a) and Eq. (4b) respectively, we obtain a relation, called it “Velocity-Acceleration-Redshift Relation”,

$$cz = v_{s1} + \frac{1}{2}a_{s1}(t_0 - t_1) \quad (5a)$$

or

$$cz = v_{s0} - \frac{1}{2}a_{s0}(t_0 - t_1). \quad (5b)$$

Eq. (5a) and (5b) are equivalent,  $v_{s1} + \frac{1}{2}a_{s1}(t_0 - t_1) = v_{s0} - \frac{1}{2}a_{s0}(t_0 - t_1)$ . Eq. (5b) can be rewritten as,

$$v_{s0} = cz + \frac{1}{2}a_{s0}(t_0 - t_1) = v_{\text{Doppler}} + \frac{1}{2}a_{s0}(t_0 - t_1). \quad (5c)$$

Eq. (5a) and (5b) imply that acceleration does shift the wavelength. The first terms on the right hand sides of Eq. (5a) and (5b) are Doppler redshift; the second terms are redshift due to acceleration, denoted as “Acceleration-Redshift”.

There are two significant predictions: first, the later we observe the wave of a source, i.e., the larger  $t_0$  is, the larger its wave redshift is; and second, the receding velocity at observing time is larger than Doppler velocity,  $v_{s0} > v_{\text{Doppler}}$ . The second prediction is important when one study the accelerating universe.

## **2.2. Acceleration-Blueshift**

When the wave source moves towards the observer with velocity  $-v_{s1}$  and constant acceleration  $-a_{s1}(= -a_{s0})$  at time  $t_1$ , the received wavelength  $\lambda_o$  at time  $t_0$  is shorter than emitted wavelength  $\lambda_s$ . Following the same argument above, we obtain “Velocity-Acceleration-Blueshift Relation”,

$$cz = -v_{s1} - \frac{1}{2}a_{s1}(t_0 - t_1), \quad (6a)$$

or

$$cz = -v_{s0} + \frac{1}{2}a_{s0}(t_0 - t_1), \quad (6b)$$

or

$$-v_{s0} = cz - \frac{1}{2}a_{s0}(t_0 - t_1). \quad (6c)$$

Eq. (6a) and (6b) are equivalent,  $-v_{s1} - \frac{1}{2}a_{s1}(t_0 - t_1) = -v_{s0} + \frac{1}{2}a_{s0}(t_0 - t_1)$ . The total blueshift is the summation of Doppler blueshifts and “Acceleration Blueshift”. Eq. (6c) shows that the forwarding velocity at observing time is larger than Doppler velocity,  $|v_{s0}| > |v_{\text{doppler}}|$ . This is important when one study the accelerating



stars/galaxies moving towards to the earth.

### **3. Deceleration Shifts Wavelength**

#### **3.1. Deceleration Reduces Doppler Redshift**

When the source recedes with velocity  $v_{s1}$  and constant deceleration  $d_{s1}$  at time  $t_1$ , we obtain “Velocity-Deceleration-Redshift Relation”,

$$cz = v_{s1} - \frac{1}{2}d_{s1}(t_0 - t_1). \quad (7a)$$

or with observed velocity  $v_{s0}$  and deceleration  $d_{s0}$  ( $= d_{s1}$ ),

$$cz = v_{s0} + \frac{1}{2}d_{s0}(t_0 - t_1), \quad (7b)$$

or

$$v_{s0} = cz - \frac{1}{2}d_{s0}(t_0 - t_1). \quad (7c)$$

Eq. (7a) and (7b) are equivalent,  $v_{s0} + \frac{1}{2}d_{s0}(t_0 - t_1) = v_{s1} - \frac{1}{2}d_{s1}(t_0 - t_1)$ . The first term of right hand side of Eq. (7a) and (7b) are Doppler redshift; the second terms imply that deceleration decreases the Doppler redshift. Eq. (7c) shows that the receding velocity at observing time is slower than Doppler velocity  $v_{s0} < v_{\text{doppler}}$ .

#### **3.2 Deceleration Reduces Doppler Blueshift**

When the source moves towards the observer with velocity  $-v_{s1}$  and constant deceleration  $d_{s1}(= d_{s0})$  at time  $t_1$ , we obtain “Velocity-Deceleration-Blueshift Relation”,

$$cz = -v_{s1} + \frac{1}{2}d_{s1}(t_0 - t_1). \quad (8a)$$

or

$$cz = -v_{s0} - \frac{1}{2}d_{s0}(t_0 - t_1), \quad (8b)$$

or

$$-v_{s0} = cz + \frac{1}{2}d_{s0}(t_0 - t_1), \quad (8c)$$

Eq. (8a) and (8b) are equivalent,  $-v_{s0} - \frac{1}{2}d_{s0}(t_0 - t_1) = -v_{s1} + \frac{1}{2}d_{s1}(t_0 - t_1)$ . The first terms of Eq. (8a) and (8b) are the Doppler blueshift; and second terms imply that deceleration reduces Doppler Blueshift. Eq. (8c) shows that the velocity at observing time is slower than Doppler velocity,  $|-v_{s0}| < |v_{\text{doppler}}|$ .

### **4. Acceleration Changes Redshift and Blueshift**

The redshift and blueshift are changed with time caused by acceleration and constant jerk,  $j_{s1} \equiv \dot{a}_{s1}$ . Take time derivative of Eq. (5a and 5b) and Eq. (6a and 6b) respectively, we conclude that acceleration and jerk cause both the change of

“Velocity-Acceleration-Redshift Relation”,

$$c\dot{z} = \frac{3}{2}a_{s1} + \frac{1}{2}\dot{a}_{s1}(t_0 - t_1), \quad (9a)$$

or

$$c\dot{z} = \frac{1}{2}a_{s0} - \frac{1}{2}\dot{a}_{s0}(t_0 - t_1), \quad (9b)$$

and the change of “Velocity-Acceleration-Blueshift Relation”,

$$c\dot{z} = -\frac{3}{2}a_{s1} - \frac{1}{2}\dot{a}_{s1}(t_0 - t_1), \quad (10a)$$

or

$$c\dot{z} = -\frac{1}{2}a_{s0} + \frac{1}{2}\dot{a}_{s0}(t_0 - t_1), \quad (10b)$$

To measure the time changes of redshift and blueshift can be used to determine the acceleration of sources.

## 5. Deceleration Changes Redshift and Blueshift

The deceleration and next order derivative cause the time changes of redshift and blueshift. Take time derivative of Eq. (7a and 7b) and Eq. (8a and 8b) respectively, we conclude that deceleration and the change of deceleration cause both the change of “Velocity-Deceleration-Redshift Relation”,

$$c\dot{z} = -\frac{3}{2}d_{s1} - \frac{1}{2}\dot{d}_{s1}(t_0 - t_1), \quad (11a)$$

or

$$c\dot{z} = -\frac{1}{2}d_{s0} + \frac{1}{2}\dot{d}_{s0}(t_0 - t_1). \quad (11b)$$

and the change of “Velocity-Deceleration-Blueshift Relation”,

$$c\dot{z} = \frac{3}{2}d_{s1} + \frac{1}{2}\dot{d}_{s1}(t_0 - t_1). \quad (12a)$$

or

$$c\dot{z} = \frac{1}{2}d_{s0} - \frac{1}{2}\dot{d}_{s0}(t_0 - t_1). \quad (12b)$$

## 6. Higher Order Derivatives Shift Wavelength

### 6.1. General Redshift

Extending Eq. (2a) to express the distance  $r$  with parameters of higher order derivatives at emitting time  $t_1$  as,

$$r = v_{s1}(t_0 - t_1) + \frac{1}{2}a_{s1}(t_0 - t_1)^2 + \frac{1}{6}j_{s1}(t_0 - t_1)^3 + \dots, \quad (13a)$$

or

$$r = \sum_{m=1}^{\infty} \frac{1}{m!} (t_0 - t_1)^m r_1^{(m)}, \quad (13b)$$

where velocity  $v_{s1} \equiv \frac{dr}{dt}\Big|_{t_1}$ , acceleration  $a_{s1} \equiv \frac{d^2r}{dt^2}\Big|_{t_1}$ , jerk  $j_{s1} \equiv \frac{d^3r}{dt^3}\Big|_{t_1}$ ,  $r_1^{(m)}$  is the

$m^{\text{th}}$  order derivative of the distance  $r$  at emitting time  $t_1$ .

Extending Eq. (2b) to express the distance  $r$  with parameters of higher order derivatives at receiving time  $t_0$  as,

$$r = v_{s0}(t_0 - t_1) - \frac{1}{2}a_{s0}(t_0 - t_1)^2 + \frac{1}{6}j_{s0}(t_0 - t_1)^3 - \dots, \quad (14a)$$

or

$$r = \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m!} (t_0 - t_1)^m r_0^{(m)}, \quad (14b)$$

where velocity  $v_{s0} \equiv \left. \frac{dr}{dt} \right|_{t_0}$ , acceleration  $a_{s0} \equiv \left. \frac{d^2r}{dt^2} \right|_{t_0}$ , jerk  $j_{s0} \equiv \left. \frac{d^3r}{dt^3} \right|_{t_0}$ ,  $r_0^{(m)}$  is the

$m^{\text{th}}$  order derivative of the distance  $r$  at receiving time  $t_0$ .

The redshift by extending Eq. (5a) with parameter at the emitting time  $t_1$  is

$$cz = v_{s1} + \frac{1}{2}a_{s1}(t_0 - t_1) + \frac{1}{6}j_{s1}(t_0 - t_1)^2 + \dots, \quad (15a)$$

or

$$cz = \sum_{m=1}^{\infty} \frac{1}{m!} (t_0 - t_1)^{m-1} r_1^{(m)}. \quad (15b)$$

The redshift in terms of parameter at receiving time is, from Eq. (5b),

$$cz = v_{s0} - \frac{1}{2}a_{s0}(t_0 - t_1) + \frac{1}{6}j_{s1}(t_0 - t_1)^2 - \dots. \quad (16a)$$

or

$$cz = \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m!} (t_0 - t_1)^{m-1} r_0^{(m)}. \quad (16b)$$

## 6.2. General Blueshift

Following the same argument above, from Eq. (6a), we obtain the extended “Velocity-Acceleration-Blueshift Relation” caused by higher order derivatives as

$$cz = -v_{s1} - \frac{1}{2}a_{s1}(t_0 - t_1) - \frac{1}{6}j_{s1}(t_0 - t_1)^2 - \dots, \quad (17a)$$

or

$$cz = -\sum_{m=1}^{\infty} \frac{1}{m!} (t_0 - t_1)^{m-1} r_1^{(m)}. \quad (17b)$$

## 7. Doppler Redshift, Cosmological Redshift, Acceleration-Redshift, and Deceleration Redshift

We have proposed the distance-redshift-acceleration-jerk relation [4] of the extended Hubble law for describing the accelerating and jerking universe, as shown below,

$$cz = \dot{r}(t_1) + \frac{1}{2H_e} \ddot{r}(t_1) + \frac{\ddot{r}(t_0)}{6H_e^2} + \dots, \quad (18)$$

where  $H_e \equiv \frac{1}{t_0 - t_1}$ .

The cosmological redshift may be expressed in Taylor series, as

$$cz = \frac{a(t_0)}{a(t_1)} - 1 = \dot{r}(t_1) + \frac{1}{2}\ddot{r}(t_1)(t_0 - t_1) + \frac{\ddot{r}(t_1)}{6}(t_0 - t_1)^2 + \dots, \quad (19)$$

where  $a(t)$  is the scale factor,  $r$  is the proper distance.

Eq. (18) and (19) are identical and have Doppler's velocity-redshift relation as the first approximation. The distance-redshift-acceleration-jerk relation and the cosmological redshift are for redshift only and limited to acceleration.

Now we have three redshift formulas: Eq. (15a) derived in this article, the distance-redshift-acceleration-jerk relation of the extended Hubble law Eq. (18) [4], and the cosmological redshift Eq. (19). The most interesting fact is that those three expressions are identical, which indicates that, indeed, acceleration shift wavelength; and that the method deriving Eq. (15a) is correct, therefore, the other conclusions derived in this article are worth to study further, although those conclusions are neither contained in the distance-redshift-acceleration-jerk relation of the extended Hubble law nor in the cosmological redshift.

## **8. Conclusion and Discussion**

We show that acceleration causes redshift, denoted as “Acceleration-Redshift”. The total redshift is a summation of Doppler redshift and Acceleration-Redshift. Moreover acceleration enlarges continuously Doppler redshift and the total redshift. When a wave source is receding with velocity and deceleration, deceleration continuously reduces Doppler redshift and the total redshift.

We have shown that when acceleration, jerk, and higher derivatives have the same direction as that of velocity, they increase Doppler shift; otherwise reduce it. Acceleration-Redshift agrees with that predicted by the cosmological redshift.

There are two significant predictions: first, the later we observe the wave, i.e., the larger  $t_0$  is, the larger the redshift is; and second, the receding velocity at observing time is larger than Doppler velocity,  $v_{s0} > v_{\text{Doppler}}$ .

Moreover we derived General Redshift and General Blueshift relations that show that not only velocity, but also acceleration, jerk, and higher order derivative motions shift wavelength. The General Redshift relation is identical with the cosmological

redshift, which implies that the cosmological redshift intrinsically includes Acceleration-Redshift. We suggest studying Acceleration-Redshift further.

We believe that to describe the accelerating universe, the redshift relation should include acceleration for accuracy and completeness.

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