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On Discrepancy in Hubble Constant

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Abstract

In the 1998, based on Supernova Ia observation, scientists report the accelerating expansion of the universe and their determination of Hubble constant. In the 2013, Planck group reports their determined value of Hubble constant based on cosmic microwave radiation. Those two different determinations lead to the discrepancy in Hubble constant. In the 2016, scientists report that the acceleration of expansion could be accelerating, denote as jerking universe, which is in debate. In the 2017, the extended Hubble law is proposed. In this article, we show that the extended Hubble law offers a theoretical approach to test the 2016 observation. Moreover, based on the extended Hubble law, we proposed a redshift-distance-velocity-acceleration-jerk relation to take into account the effects of acceleration and jerk, and show that the measured value from Supernova Ia observation is actually the value of the effective Hubble constant that is greater than the value of the regular Hubble constant and, thus provides an explanation of the discrepancy in Hubble constant.

Key words: Hubble constant, redshift, discrepancy, Supernova Ia,

Cosmic microwave radiation, cosmology,

1. Introduction

Hubble law plays a fundamental role in cosmology. One way to determine Hubble Constant that represents the expansion rate of the universe is to measure the redshift of Supernova Ia by utilizing the distance modulus [1, 2]. Another way to determine the rate of expansion is from the microwave background radiation [3]. For a flat universe, so determined Hubble constants are expected to be the same. However Hubble constant obtained from Supernova Ia is greater than that obtained from Planck mission, which is a discrepancy.

Moreover, in the 2016, Scientists report that the acceleration of the expansion of the universe is faster than expected [2], denote as the jerking universe. There is a debate on the 2016 observation. If the 2016 observation is confirmed, one faces the following tasks: dynamically, since the existing theories of gravity are second order derivative, there is lack of both a mechanism and a higher order derivative theory of gravity; kinematically, there is lack of both a general distance-motion relation and a redshift-distance-motion relation to reflect the effects of acceleration and jerk. In the 2017, the extended Hubble law is proposed to describe the effects of acceleration and jerk [4].

We argue that above discrepancy and tasks may relate to Hubble law. In this article we focus on kinematics and applies the extended Hubble law to offer a resolution for those two issues.

2. Review of Extended Hubble Law

In the 1929, Hubble proposed the distance-velocity relation, $\dot{r} = Hr$, or rewrite as

$$r = \frac{1}{H}\dot{r} \qquad \text{or} \qquad H \equiv \frac{\dot{r}}{r}, \tag{1}$$

to describe the uniformly expanding universe. We argue that it should be extended to describe the effects of accelerating and jerking, respectively. In this section let's review the extended Hubble law that includes the distance-velocity-acceleration relation and distance-velocity-acceleration-jerk relation [4].

The proposed distance-velocity-acceleration-jerk relation is,

$$r(t_1) = \alpha\dot{r}(t) + \beta\ddot{r}(t) + \gamma\dddot{r}(t). \tag{2}$$

Based on kinematics and Hubble law, we assume,

$$\alpha \equiv \frac{1}{H_e}, \qquad \beta \equiv \frac{1}{2!H_e^2}, \qquad \gamma \equiv \frac{1}{3!H_e^3},$$

where the *extended Hubble parameter* is defined as (time t is prior to time t_1),

$$H_e \equiv \frac{1}{t_1 - t}. \tag{3}$$

Eq. (2) is the *extended Hubble law* and may be utilized to describe different universes as below,

(A) For a uniformly expanding universe: $$r(t_1) = \frac{1}{H_e}\dot{r}(t), \tag{4}$$

(B) For an accelerating universe: $$r(t_1) = \frac{1}{H_e}\dot{r}(t) + \frac{1}{2!}\frac{1}{H_e^2}\ddot{r}(t), \tag{5}$$

(C) For a jerking universe: $$r(t_1) = \frac{1}{H_e}\dot{r}(t) + \frac{1}{2!}\frac{1}{H_e^2}\ddot{r}(t) + \frac{1}{3!}\frac{1}{H_e^3}\dddot{r}(t). \tag{6}$$

For a higher order expanding universe, we propose a general distance-motion relation:

$$r(t_1) = \sum_{n=1}^{\infty}\frac{1}{n!}\frac{1}{H_e^n} r^{(n)}(t). \tag{7}$$

Where, superscript (n) denote n[th] order derivative with respective to time.

For a uniformly expanding universe, $H_e = \frac{\dot{r}(t)}{r(t_1)} = H$ and Eq. (2 and 4) is equivalent to Hubble law. Eq. (2, 4-6) show that when apply the regular Hubble parameter H in studying a non-uniformly expanding universe, the effects of acceleration and jerk are neglected.

For an accelerating universe, Eq. (5) gives the extended Hubble parameter,

$$H_e = \frac{1}{2}\frac{\dot{r}(t)}{r(t_1)}\left\{1 + \sqrt{1 + 2\frac{\ddot{r}(t)r(t_1)}{\dot{r}^2(t)}}\right\} = \frac{1}{2}H\left\{1 + \sqrt{1 + 2\frac{\ddot{r}(t)r(t_1)}{\dot{r}^2(t)}}\right\}. \tag{8}$$

For a jerking universe, Eq. (6) gives the extended Hubble parameter,

$$H_e = \frac{1}{3}\frac{\dot{r}(t)}{r(t_1)}\left\{1 + \sqrt[3]{1 + \frac{A}{y^3} + \sqrt{\left(1 + \frac{A}{y^3}\right)^2 - \left(1 - \frac{w}{y^2}\right)^3}} + \right.$$

$$\left. + \sqrt[3]{1 + \frac{A}{y^3} - \sqrt{\left(1 + \frac{A}{y^3}\right)^2 - \left(1 - \frac{w}{y^2}\right)^3}}\right\}, \tag{9}$$

where $y = \frac{\dot{r}(t)}{3r(t_1)}$, $w = -\frac{1}{6}\frac{\ddot{r}(t)}{r(t_1)}$, $A = \frac{1}{12}\frac{\dot{r}(t)\ddot{r}(t)}{r^2(t_1)} + \frac{1}{12}\frac{\dddot{r}(t)}{r(t_1)}$.

Moreover, Eq. (5-7) show that, for a non-uniformly expanding universe, there is not only the linear distance-velocity relation but also linear distance-acceleration relation, and linear distance-jerk relation. If the 2016 observation is confirmed, velocity $\dot{r}(t)$, acceleration $\ddot{r}(t)$, and jerk $\dddot{r}(t)$ should be determined dynamically by a third order derivative theory of gravity.

With the extended Hubble parameter, let's extend the deceleration parameter q to the "*motion parameter*", that judges the motion status at time t, and is defined as,

$$q_n \equiv -\frac{r^{(n)}(t)}{r(t_1)H_e^n}. \tag{10}$$

With Eq. (10), Eq. (6) may be rewritten in different forms as,

$$1 = -q_1 - \frac{1}{2}q_2 - \frac{1}{3!}q_3, \tag{6a}$$

$$\frac{\dot{r}(t)}{r(t_1)} = H_e(1 + \frac{1}{2}q_2 + \frac{1}{3!}q_3) \equiv \overline{H}_e, \tag{6b}$$

$$\frac{\ddot{r}(t)}{r(t_1)} = 2H_e^2\left\{1 + q_1 + \frac{1}{3!}q_3\right\}, \tag{6c}$$

$$\frac{\dddot{r}(t)}{r(t_1)} = 3!\,H_e^3\left(1 + q_1 + \frac{1}{2}q_2\right), \tag{6d}$$

and Eq. (7) gives,

$$\frac{r^{(n)}(t)}{r(t_1)} = n!\,H_e^n\left(1 + \sum_{m\neq n}^{\infty}\frac{1}{m!}q_m\right). \tag{11}$$

Where $\overline{H}_e$ is the effective extended Hubble parameter.

Now let's convert parameters $r^{(n)}(t)$ to $r^{(n)}(t_1)$. Defining the "same-time motion parameter" q_{sn}, which judging the motion status at the same time t_1, as

$$q_{sn} \equiv -\frac{r^{(n)}(t_1)}{r(t_1)H_e^n}. \tag{12}$$

The distance-velocity-acceleration-jerk relation, Eq. (6), becomes,

$$r(t_1) = \dot{r}(t_1)\left(\frac{1}{H_e}\right) - \frac{1}{2!}\ddot{r}(t_1)\left(\frac{1}{H_e}\right)^2 + \frac{1}{3!}\dddot{r}(t_1)(\frac{1}{H_e})^3, \tag{6s}$$

$$1 = -q_{s1} + \frac{1}{2}q_{s2} - \frac{1}{6}q_{s3}, \tag{6sa}$$

$$\frac{\dot{r}(t_1)}{r(t_1)} = H_e\left\{1 - \frac{1}{2}q_{s2} + \frac{1}{6}q_{s3}\right\} \equiv \overline{H}_{se}, \tag{6sb}$$

$$\frac{\ddot{r}(t_1)}{r(t_1)} = -2H_e^2\left\{1 + q_{s1} + \frac{1}{6}q_{s3}\right\}, \tag{6sc}$$

$$\frac{\dddot{r}(t_1)}{r(t_1)} = 6H_e^3\left\{1 + q_{s1} - \frac{1}{2}q_{s2}\right\}. \tag{6sd}$$

We suggest that, in the study of the accelerating/jerking universe, replace $\frac{\dot{r}(t)}{r(t_1)}$ and $\frac{\dot{r}(t_1)}{r(t_1)}$ by $\overline{H}_e$ and $\overline{H}_{se}$, respectively, instead by the regular Hubble parameter $H = \frac{\dot{r}}{r}$.

3. Independent Test of the 2016 Observation

There is a debate on the 2016 observation. Besides pursuing the observational confirmation, the extended Hubble law (Eq. (6)) provides a theoretical test, i.e., the extended distance-motion relation offers an independent approach to test the validation of the 2016 observation.

Once q_1 and q_2 or q_{s1} and q_{s2} are determined, Eq. (6d and 6sd) indicate that as long as either

$$1 + q_1 + \frac{1}{2}q_2 \neq 0, \tag{13}$$

or

$$1 + q_{s1} - \frac{1}{2}q_{s2} \neq 0, \tag{14}$$

there is inevitably a jerk, $\dddot{r}(t) \neq 0$, otherwise there will be no jerk.

The significance of this approach is that the confidence level of the conclusion on the existence of jerk is the same as that of measurements of velocity and acceleration.

4. Extending Distance-Redshift Relation

For offering an explanation of the discrepancy in Hubble constant, in this section, we extend the regular distance-redshift relation,

$$H_0 d_p = cz, \tag{15}$$

to study the redshift phenomena of the acceleration and jerk universe. For this aim, A simplest way is to extend it to a distance-redshift-velocity-acceleration-jerk relation by substitute the distance-motion relation, Eq. (6sb), into Eq. (15). We obtain different extended distance-redshift relations for different universes:

(1) For a uniformly expanding universe: $H_e d_p = cz$; $(H_e = H)$ (16)

(2) For a uniformly accelerating universe: $H_e\left\{1 - \frac{1}{2}q_{s2}\right\} d_p = cz$; (17)

(3) For a uniformly jerking universe: $$H_e\left\{1-\frac{1}{2}q_{s2}+\frac{1}{6}q_{s3}\right\}d_p = cz. \quad (18)$$

Eq. (17 and 18) represent the effects of acceleration and jerk on redshift, respectively, which are to enlarge the redshift.

The extended distance-redshift relation provides a kinematical approach to explain the relation between observed result and Hubble constant.

5. Explaining Discrepancy in Hubble Constant

For this aim, let's study the measurement of the redshift of Supernova Ia. It starts with the distance modulus,

$$m - M = 5\log(d) + 25. \quad (19)$$

For different universes, we obtain different redshift-magnitude relation as below,

(A) For a uniformly expanding universe, Eq. (16 and 19) give,

$$m = M + 5\log(cz) - 5\log[H_0] + 25; \quad (20a)$$

(B) For a uniformly accelerating universe, Eq. (17 and 19) give,

$$m = M + 5\log(cz) - 5\log\left[H_0\left(1-\frac{1}{2}q_{s2}\right)\right] + 25; \quad (20b)$$

(C) For a uniformly jerking universe, Eq. (18 and 19) give,

$$m = M + 5\log(cz) - 5\log\left[H_0\left(1-\frac{1}{2}q_{s2}+\frac{q_{s3}}{3!}\right)\right] + 25. \quad (20c)$$

Where we have used an approximation, $\overline{H}_{se0} \approx H_0$.

Eq. (20a) has been used to determine Hubble constant conventionally for accelerating and jerking universes. We argue that this is the root cause of the discrepancy as shown bellow. Based on Eq. (20b and 20c), for the accelerating and jerking universes, the values one obtained from the 1998 and the 2016 observations are actually, respectively,

$$H_{1998} \equiv H_0\left(1-\frac{1}{2}q_{s2}\right), \quad (21)$$

$$H_{2016} \equiv H_0\left(1-\frac{1}{2}q_{s2}+\frac{q_{s3}}{3!}\right), \quad (22)$$

which include the effects of acceleration q_{s2} and jerk q_{s3}.

For an accelerating universe, $q_{s2} < 0$, so the regular Hubble constant H_0 is smaller than H_{1998}, similarly, smaller than H_{2016}.

Therefore the value of Hubble constant H_0 obtained from other measuring method is smaller than that obtained from the distance modulus of Supernova Ia, which provides an explanation for the discrepancy in Hubble constant.

6. Evolution of Redshift

The drift of redshift can be used to measure cosmographic parameters before a dynamic theory is established. The evolution of redshift needs to take into account the effects of acceleration and jerk. Let's start from the cosmological redshift,

$1 + z = \frac{r(t_0)}{r(t)}$. Taking derivative, we obtain,

$$\frac{dz}{dt_0} = H_e(t_0)\left[1 - \frac{1}{2}q_{s20} + \frac{1}{6}q_{s30}\right](1 + z) - H_e(t)\left[1 - \frac{1}{2}q_{s2} + \frac{1}{6}q_{s3}\right]. \quad (23)$$

Once we find the change of redshift, we may obtain the extended Hubble parameter at a prior time t from,

$$H_e(t) = H_e(t_0)(1 + z)\left[\frac{1-\frac{1}{2}q_{s20}+\frac{1}{6}q_{s30}}{1-\frac{1}{2}q_{s2}+\frac{1}{6}q_{s3}}\right] - \frac{dz}{dt_0}\left[\frac{1}{1-\frac{1}{2}q_{s2}+\frac{1}{6}q_{s3}}\right]. \quad (24)$$

7. Summary

We propose a redshift-distance-velocity-acceleration-jerk relation. Based on this relation, it is shown that the measured value from Supernova Ia observation is actually the value of the effective extended Hubble constant that is greater than the value of the regular Hubble constant. Thus it provides an explanation of the discrepancy in Hubble constant.

Moreover, we show that the extended Hubble law offers an independent theoretical approach to test the 2016 observation. This approach converts the confidence level of the conclusion on the existence of jerking expansion to that of measurements of uniform expansion and accelerating expansion, the latter are commonly accepted.

Based on the extended Hubble law, the drift of redshift is expressed in terms of velocity, acceleration, and jerk.

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